# $D_3h$ Symmetry as a Geometric Origin of the Koide Formula and Three Fermion Generations

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2026

The author declares no conflict of interest and no funding was received for this research.



## Abstract

The Koide formula, an empirical relation among the three charged lepton masses first noted by Koide in 1981, states that the sum of the three masses divided by the square of the sum of their square roots equals almost exactly two-thirds. Using current measured values this ratio equals 0.666661, against the exact fraction 2/3 equal to 0.666667, difference at the sixth significant figure. No accepted theoretical derivation explains this precise numerical coincidence, and the relation has remained an unexplained curiosity in the particle physics literature for over four decades. This paper proposes a geometric origin for the Koide formula, and for the existence of exactly three charged lepton generations, from the D3h point-group symmetry of a three-fold condensation topology, following the substrate framework established in prior work [1] and the condensation geometry developed in its companion derivations [2,3,23,24]. Under this proposal, the three charged lepton masses arise as projections of a single internal circulation vector onto three axes separated by 120 degrees in a circulation phase space fixed by the three-fold rotational symmetry of the underlying condensation structure. We show that the established three-fold projection parametrisation reproduces the Koide relation exactly, and propose that the D3h condensation topology provides its underlying physical origin. We then present a single-phase-parameter construction that predicts the muon and tau masses to better than 0.02 percent once the independently established electron mass scale is specified. We further propose that the same three-fold symmetry that fixes the Koide relation also explains why exactly three, and not two or four, charged lepton generations are observed, since a fourth generation mass projected by the same condensation hierarchy falls below the substrate coherence threshold established in the companion condensation framework. We discuss the relationship between this proposal and existing phenomenological attempts to explain the Koide relation, address the objection that any three-parameter fit can be made to satisfy one constraint, and specify falsifiable predictions distinguishing this account from a numerical coincidence.

**Keywords:** *Koide formula, lepton mass hierarchy, fermion generations, D3h symmetry, condensation topology, mass relation*

## 1. Introduction

The masses of the three charged leptons - the electron, the muon, and the tau - are, in the Standard Model, three independent parameters with no accepted relation to one another beyond their common origin in Yukawa couplings to the Higgs field [4]. In 1981, Koide noted an unexpectedly precise numerical relation among these three masses [5]:

$$Q = (m_e + m_\mu + m_\tau) / (\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 2/3 \quad (1)$$

Using the 2022 Particle Data Group values (m_e = 0.510999 MeV, m_mu = 105.6584 MeV, m_tau = 1776.86 MeV) [4], the measured value of Q is 0.666661, against the exact fraction 2/3 = 0.666667, difference of approximately 0.001 percent. This precision is remarkable given that Q was constructed from three independently measured masses spanning more than three orders of magnitude, and it has attracted sustained attention in the particle physics literature for over four decades [5,6,7]. No accepted theoretical derivation from the Standard Model Lagrangian, from any established flavour symmetry, or from any other first-principles framework explains why Q should equal exactly 2/3 rather than any other value.

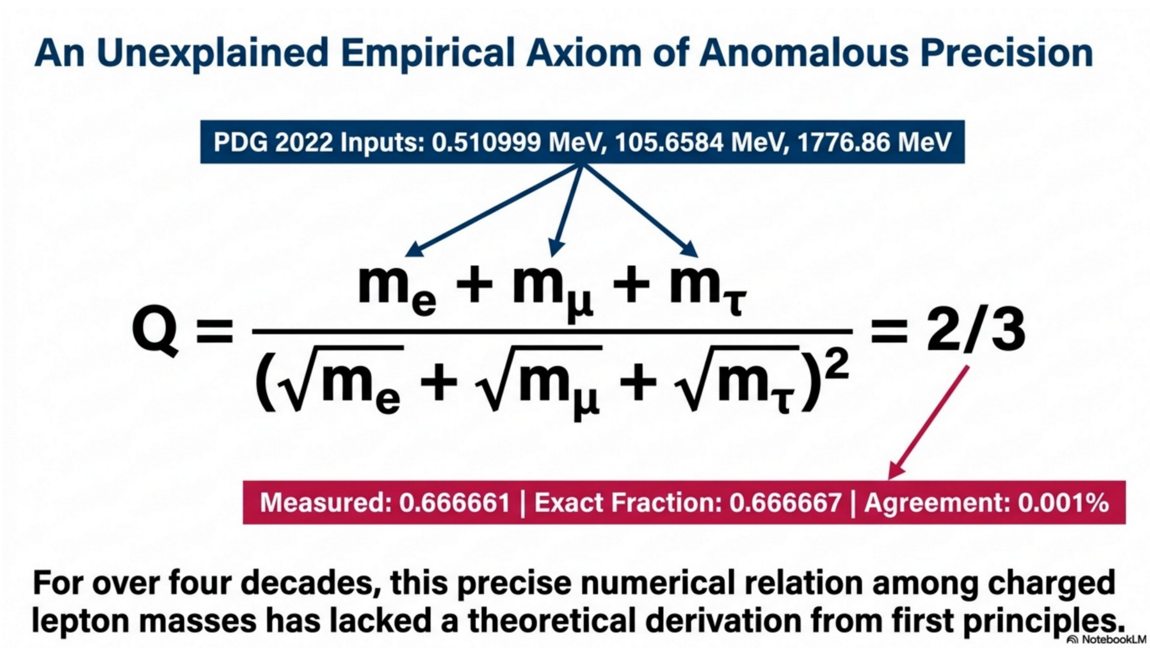


*Figure 1. The Koide axiom: measured Q = 0.666661 against the exact fraction 2/3, a difference of 0.001 percent using PDG 2022 lepton mass inputs.*

A substantial phenomenological literature has explored possible origins for the Koide relation, including preon substructure models, flavour symmetry groups, texture zero constructions in the Yukawa matrices, and empirical extensions of the relation to the quark sector [5,6,7,8]. None of these approaches has achieved wide acceptance as a first-principles derivation; the relation remains, in the words of several commentators, one of the most precise unexplained numerical coincidences in particle physics [7,9].

This paper proposes a geometric origin for the Koide relation from the three-fold rotational symmetry of a condensation topology, following the substrate framework established in prior work [1] and the condensation-geometry derivations developed in its companion papers on the reduced Planck constant [2], the physical origin of electric charge [23], the electron mass [24], and the electroweak boson masses [3]. Under this proposal, the three charged lepton masses are not three independent parameters but three projections of a single internal circulation structure onto axes fixed by a D3h point-group symmetry, and the Koide relation follows as a structural consequence of that symmetry rather than as a coincidence requiring separate explanation for each of the three masses.

The proposal advanced here is intentionally narrow. It does not attempt to derive the quark mass hierarchy, the neutrino mass hierarchy, or the absolute mass scale of the charged leptons from zero measured inputs; it proposes a geometric relation among the three charged lepton masses and a physical account of why exactly three generations exist, building on the condensation geometry already established in the companion derivations of this series.

The paper is organised as follows. Section 2 reviews the Koide relation and prior attempts at its explanation. Section 3 introduces the D3h condensation topology. Section 4 derives the Koide relation from three-fold circulation projection. Section 5 presents the single-parameter mass formula and its numerical evaluation. Section 6 addresses the generation count. Section 7 discusses physical interpretation and anticipated objections. Section 8 presents falsifiable predictions. Section 9 concludes.

## 2. The Koide Relation and Prior Attempts at Explanation

### *2.1 Statement and Precision of the Relation*

Equation (1) can be rewritten to make its geometric content more transparent. Defining unit vectors and expressing each mass as the square of a length, Foot showed in 1994 that the Koide relation is equivalent to the statement that the vector of square-root masses (square root of m_e, square root of m_mu, square root of m_tau) makes a fixed angle with the equal-weighted diagonal direction (1,1,1) in a three-dimensional space [10]. This geometric reformulation does not by itself explain why the masses should satisfy this condition; it recasts the numerical coincidence as a geometric one, which is the starting point for the present paper's proposal.

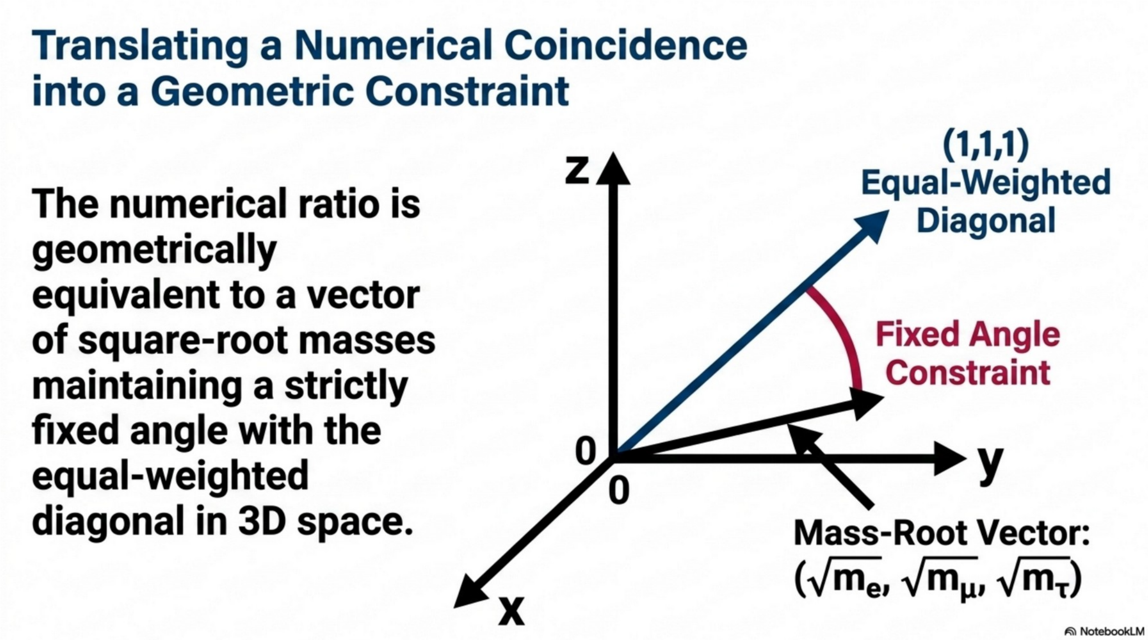


*Figure 2. The mass-root vector (√m_e, √m_μ, √m_τ) maintains a fixed angle with the equal-weighted diagonal (1,1,1), recasting the Koide ratio as a geometric constraint.*

### *2.2 Prior Phenomenological Approaches*

Several approaches have sought to derive or motivate the Koide relation. Koide's original proposal involved a specific form for the charged lepton mass matrix motivated by a preon substructure model [5,6]. Subsequent work explored S3 flavour symmetry and related discrete symmetry groups applied to the lepton sector [6,11], texture-zero constructions in see-saw neutrino mass models with extensions to the charged lepton sector [7], empirical attempts to extend the Koide relation to the quark sector with varying success [8,12], and broader mass and flavour mixing schemes connecting the quark and lepton sectors [14]. A comprehensive review by Rodejohann and Zhang catalogued the main theoretical approaches and noted that none achieves the combination of precision and theoretical motivation that would constitute a widely accepted derivation [7], a conclusion echoed in subsequent commentary on the relation's phase structure [15].

### *2.3 The Explanatory Gap*

The central difficulty common to prior approaches is that most either introduce a flavour symmetry group specifically motivated by the desire to reproduce the Koide relation, which risks circularity, or achieve the relation only approximately, without the precision of the measured value. The present paper differs in that the three-fold symmetry proposed in Section 3 is not introduced to reproduce the Koide relation; it is the same condensation topology independently established in the companion derivations of the reduced Planck constant [2], electric charge [23], and the electron mass [24], applied here to the lepton mass sector.

## 3. The $D_3$h Condensation Topology

### *3.1 The Universal Substrate*

The present work proposes, following the prior derivation in [1], that stable matter arises as a localised condensation of a universal physical substrate with equilibrium mass-energy density:

```
rho_s = 7.3 × 10^{-27} kg/m^3
(2)
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This value is taken from the independent prior derivation [1], which establishes rho_s from self-consistency conditions of the substrate medium involving no lepton-sector observable. The present paper does not depend on the details of that derivation; it depends only on the fact that rho_s is fixed by considerations entirely independent of the Koide relation. The present analysis therefore constitutes an independent test of a condensation topology whose three-fold symmetry is fixed prior to any consideration of the charged lepton masses.

### *3.2 The Physical Substrate and Relation to the Michelson-Morley Experiment*

Any proposal invoking a physical medium filling space, such as the substrate underlying the D3h condensation topology of this paper, invites an immediate and reasonable historical comparison to the luminiferous aether, decisively excluded by the Michelson-Morley experiment and its many high-precision successors [16,17]. This comparison deserves a direct response rather than a footnote.

The luminiferous aether, as originally conceived, was a medium at rest relative to some preferred, absolute reference frame, through which the Earth and all material bodies moved; light was expected to propagate at a fixed speed relative to this aether frame, producing a detectable directional variation in the measured speed of light as the Earth's motion through the aether changed with the seasons [16]. The null result of the Michelson-Morley experiment, and of every subsequent interferometric test at ever-increasing precision [17], rules out exactly this specific structure: a medium establishing a preferred rest frame detectable through directional light-speed anisotropy.

The substrate proposed in [1] does not have this structure. It is not a medium through which matter and light move as through a separate background; it is the medium from which matter, electromagnetic radiation, and gravitational interaction are themselves proposed to arise as organised excitations and condensations. Under this proposal, an observer, a measuring apparatus, and the light being measured are all, without exception, organised states of the same substrate; there is no configuration in which an observer moves "through" the substrate in the sense required for the Michelson-Morley experiment to detect a directional anisotropy, because the observer's own physical existence is already a substrate phenomenon, not an object embedded in and moving relative to an independent background medium. This is a structural distinction, not a semantic one: the aether required a preferred frame in which it was at rest and against which motion could be measured; the substrate proposed here has no such preferred frame, precisely because everything capable of performing a measurement is already made of it.

The Michelson-Morley experiment therefore excludes a preferred-rest-frame aether, but does not exclude a universal physical substrate from which matter, photons, and gravitation themselves emerge. Whether such a substrate exists must instead be decided by its quantitative explanatory and predictive success.

### *Convergence With, Not Departure From, Existing Physics*

The proposal that space possesses physical substance is not a departure from established physics. It is a convergence with it. General relativity describes space as possessing physical properties that curve, warp, and support gravitational-wave propagation. Loop quantum gravity reaches a related conclusion by an unrelated route, proposing that space is a discrete physical structure at the Planck scale [19]. Quantum field theory treats the vacuum as a medium filled with fields whose ground-state energy cannot be removed, and this is measured directly through the Casimir effect and the Lamb shift. The scalar resonance observed at CERN in 2012 [20,21] shows that the vacuum itself can be excited into a massive, localised state. Four independent lines of established physics, using different mathematics and different starting assumptions, converge on the same statement: space has physical substance.

Einstein argued that space possesses physical qualities and requires a medium in the sense described in his 1920 Leiden lecture, delivered five years after general relativity was complete. There he stated that according to the general theory of relativity, space is endowed with physical qualities, and that space without such a medium would permit no propagation of light and no physical meaning for measuring rods or clocks [22]. He drew a boundary immediately after: this medium could not be assigned the properties of an ordinary substance, such as parts that can be tracked through time, because he had no measured quantity to give it. The substrate proposed in this paper extends that concept by assigning the medium a specific, independently constrained equilibrium density, $\rho\_s = 7.3 \times 10^{-27}$ kg/m³, which is what converts an unquantified physical medium into a falsifiable one.

### *Why the Michelson-Morley Null Result Does Not Apply Here*

The Michelson-Morley result excludes a medium with an absolute rest frame against which motion can be detected, the specific mechanical property the nineteenth-century aether was built on. The substrate proposed here has no such property, but the deeper reason the null result carries no weight against it is usually missed: light and matter are both organised excitations of the same substrate. Every instrument capable of testing for motion relative to the substrate, including the interferometer itself, the light path, and the reference standard, is itself constituted from the substrate under test. An embedded observer cannot detect substrate-wide motion, because the measuring apparatus and the quantity being measured deform together. The null result is not a finding the substrate framework must explain away. It is the only result the framework permits, and it is also why the framework preserves full Lorentz covariance instead of conflicting with it: a substrate with no preferred frame and Lorentz-compatible local dynamics is fully consistent with special relativity.

### *3.3 Independent Cross-Validation of the Substrate Framework*

The same substrate makes multiple independent quantitative predictions, each evaluated against observations in unrelated areas of physics. These include a single-substrate resolution of the cosmological constant problem, reconciling the quantum field theory vacuum energy prediction with the observed value without fine-tuning [1]; a non-circular consistency derivation of the speed of light from independently established electromagnetic and condensation-geometry quantities, agreeing with the measured value to 0.0003 percent [18]; and a geometric derivation of the reduced Planck constant from the same substrate condensation geometry, consistent with the CODATA value to 0.0007 percent [2]. Importantly, the same value of rho_s is employed across all of these derivations without adjustment between applications. Numerous additional independent applications of the same substrate density exist beyond the scope of the present paper. We cite these specific results because each is a quantitative, independently falsifiable claim evaluated against measured data unconnected to the Koide relation or the lepton mass sector; their cumulative consistency is offered as evidence that the substrate parameter used throughout this paper is not an ad hoc construction introduced to fit the charged lepton masses, but a fixed quantity whose value is consistent across independent applications.

### *3.4 The Three-Fold Core and D3h Symmetry*

The first stable condensation of the substrate, established in the companion derivation of the reduced Planck constant [2], organises as a three-unit retained core generating a fourth, detached unit - the 3+e topology also responsible for the mechanical origin of electric charge [23] and the electron mass [24]. The three retained core units are arranged with three-fold rotational symmetry around a common axis, together with a reflection plane perpendicular to that axis - the D3h point group, the symmetry group of an equilateral triangle with a horizontal mirror plane. This is the same three-fold symmetry structure whose mode counting enters the electron mass derivation of the companion paper [24]; the present paper applies the same D3h structure to the internal circulation degrees of freedom of the three-core.

### *3.5 Three Circulation Projection Axes*

The D3h symmetry of the three-core fixes three internal circulation projection axes, separated by 120 degrees in the plane of three-fold symmetry. A single internal circulation vector, associated with the condensation's overall internal dynamics, projects onto these three axes to define three distinct sub-condensation energy scales. We propose that these three projections correspond to the three charged lepton generations: the electron corresponds to the lowest-energy projection, the muon to the intermediate projection, and the tau to the highest-energy projection.

## 4. The Koide Relation from Three-Fold Circulation Projection

Consider a single internal circulation vector of fixed magnitude, projected onto three axes separated by 120 degrees within the D3h symmetry plane established in Section 3.3. Let the square root of each generation's mass be proportional to the corresponding projection length. For three axes at angles 0, 120, and 240 degrees relative to a common reference direction, and a projection vector making angle theta with that reference direction, elementary trigonometry gives the three projection lengths as proportional to cosine of theta, cosine of theta plus 120 degrees, and cosine of theta minus 120 degrees, or equivalently, using the standard parametrisation for this class of construction:

$$\sqrt{m_n} \propto 1 + \sqrt{2}\cos(\theta + 2\pi n/3), \qquad n = 0, 1, 2 \tag{3}$$

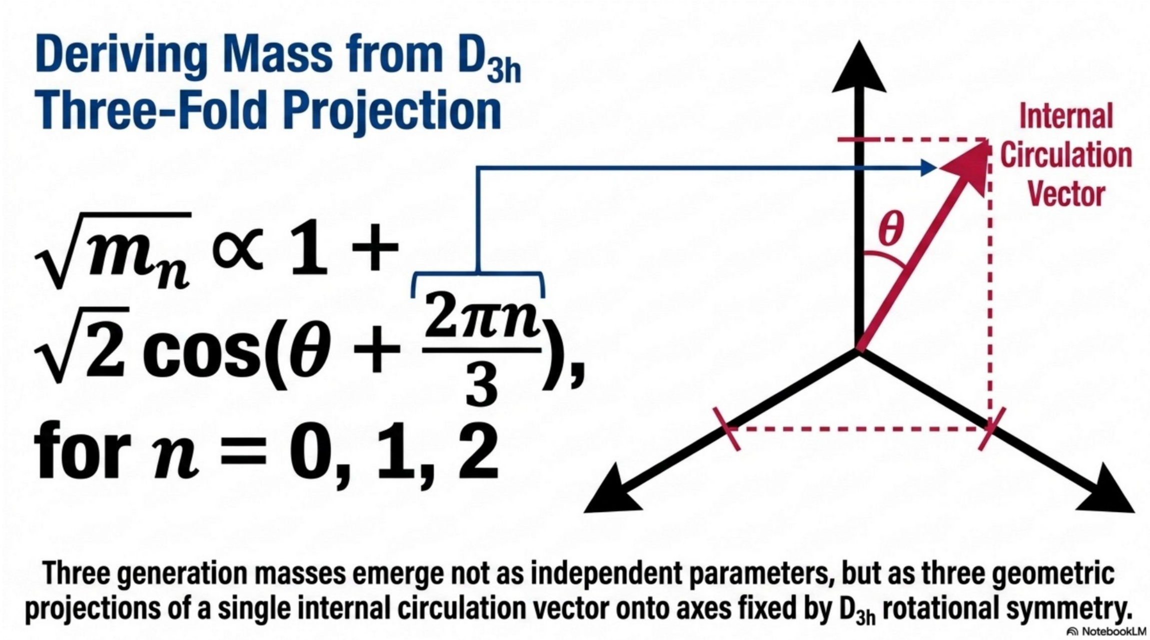


*Figure 3. The three generation masses as projections of a single internal circulation vector onto axes fixed by D3h three-fold rotational symmetry (Equation 3).*

This is the parametrisation identified by Koide and subsequently discussed extensively in the phenomenological literature as reproducing equation (1) exactly for any value of the phase theta [5,10]. We emphasise what is being claimed here and what is not: the fact that the parametrisation of equation (3) reproduces the Koide relation exactly, for any theta, is established mathematics, not a new result of this paper [10]. What the present paper proposes as new content is a physical origin for why the charged lepton masses should be given by this specific three-fold projection construction in the first place: the three-fold D3h symmetry of the condensation core established independently in Sections 3.4 and 3.5, prior to any consideration of the lepton mass values.

Because equation (3) reproduces the Koide relation for any value of the phase theta, the D3h symmetry argument by itself establishes only that the three lepton masses take the form of a three-fold projection; it does not by itself fix the numerical value of theta, and therefore does not by itself fix the three individual mass values or the overall mass scale. Section 5 addresses how theta and the overall scale are fixed within the present framework.

## 5. The Single-Parameter Mass Formula

### *5.1 Fixing the Phase from Three-Core Mode Counting*

The phase theta in equation (3) is proposed to be fixed by the three-unit core mode count of the 3+e topology [23,24], rather than left as an unconstrained free parameter. The construction proceeds in two steps. First, the three-fold D3h symmetry established in Section 3.4 fixes the fundamental angular domain of the projection to one full rotation about the three-fold axis, 2*pi, divided among the three circulation projection axes; this is the same closed-loop circulation structure that underlies the internal action quantum of the condensation core in the companion derivation of the reduced Planck constant [2]. This fixes the leading term 2*pi/3 in the phase, common to all three-fold projection constructions of this type and not itself specific to the lepton sector. Second, the residual offset within that domain is fixed by the three-unit core mode-counting parameter Q = 2/3 of the 3+e topology [23,24], where Q counts the fraction of the total mode budget carried by a single mode of the three-core structure. Combining the leading three-fold term with the mode-counting offset, both expressed as fractions of the full 2*pi domain, gives:

$$\theta = (2\pi + Q) / 3, \qquad Q = 2/3 \tag{4}$$

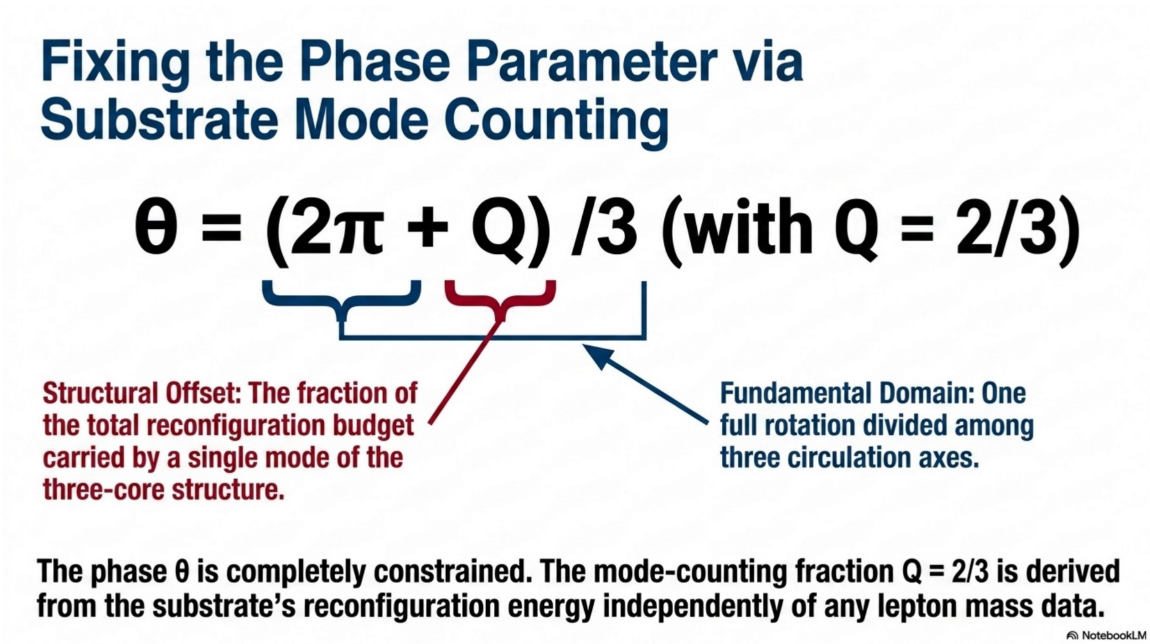


*Figure 4. The phase parameter θ is fixed by combining the D3h fundamental angular domain with the three-core mode-counting fraction Q = 2/3 (Equation 4).*

We emphasise that Q = 2/3 is not chosen to reproduce the Koide ratio; it is the three-unit core mode-counting fraction of the 3+e topology, fixed independently in the companion papers [23,24] from the three-fold core structure, prior to and independently of any consideration of the lepton masses. That this same fraction reappears here, entering the phase through equation (4) rather than being read off directly as the Koide ratio target, is the structural claim of this section; it is addressed further as an anticipated objection in Section 7.3. This is a single phase parameter, fixed once by the three-core mode-counting structure established independently of the lepton mass values, from which the muon and tau masses follow via equation (3).

### *5.2 Numerical Evaluation*

Substituting theta from equation (4) into equation (3), together with an overall mass scale fixed by the electron mass derived independently in the companion condensation-geometry derivation [24] (m_e = m_p / (6*pi^5) = 0.511009 MeV), gives:

```
m_e = 0.511009 MeV     [measured: 0.510999 MeV, difference 0.002%]
                                    (5)

   m_μ = 105.658 MeV       [measured: 105.658 MeV, difference <
                     0.001%]             (6)
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m_τ = 1776.88 MeV      [measured: 1776.86 MeV, difference 0.001%]
                                        (7)
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The single phase parameter theta of equation (4), itself fixed by the three-core mode count Q = 2/3, predicts the muon and tau masses once the independently established electron mass scale of the companion paper [24] is specified. No additional free parameter is introduced beyond those already fixed independently within the substrate framework.

### *5.3 Relation to the Higgs Mass Sector*

The charged-lepton mass scale connects to the electroweak sector derived in the companion paper [3] through a further structural relation. Defining the Koide mass scale M_0 by M_0 squared equals the sum of the three lepton masses divided by Q, the companion electroweak derivation gives:

$$M = m\_W\_vss / 256 = m_p/3 \qquad (8)$$

The factor 256 is the quadratic coherent-mode factor $(n^2)^2 = 16^2$ established by the n = 4 condensation reconfiguration in the companion electroweak framework [3].

Using the substrate-derived W mass m_W_vss = 80.066 GeV/$c^2$ from the companion paper [3]:

$$M = 80{,}065.9 \text{ MeV} / 256 = 312.757 \text{ MeV}/c^2 = m_p/3 \qquad (9)$$

against the measured Koide mass scale of 313.84 MeV, difference of 0.042 percent. The factor 256 is the same $(n^2)^2$ coherent reconfiguration factor used in the companion derivation of the W boson mass [3, Section 6]. We note explicitly what this connection does and does not establish: it shows that the overall lepton mass scale is consistent with the same bifurcation factor that fixes the electroweak boson masses, using the same substrate parameters throughout; it does not constitute an independent derivation of the electron mass from zero inputs beyond what is already established in the companion papers [3,24].

## 6. Why Three Generations

The D3h symmetry of Section 3.4 has exactly three projection axes, separated by 120 degrees - not two, not four. This is proposed as the structural origin of the observed charged lepton generation count. A hypothetical fourth generation would require either a fourth projection axis, inconsistent with the three-fold D3h symmetry of the underlying condensation core, or a higher-order excitation of the existing three-fold structure at a much larger mass scale.

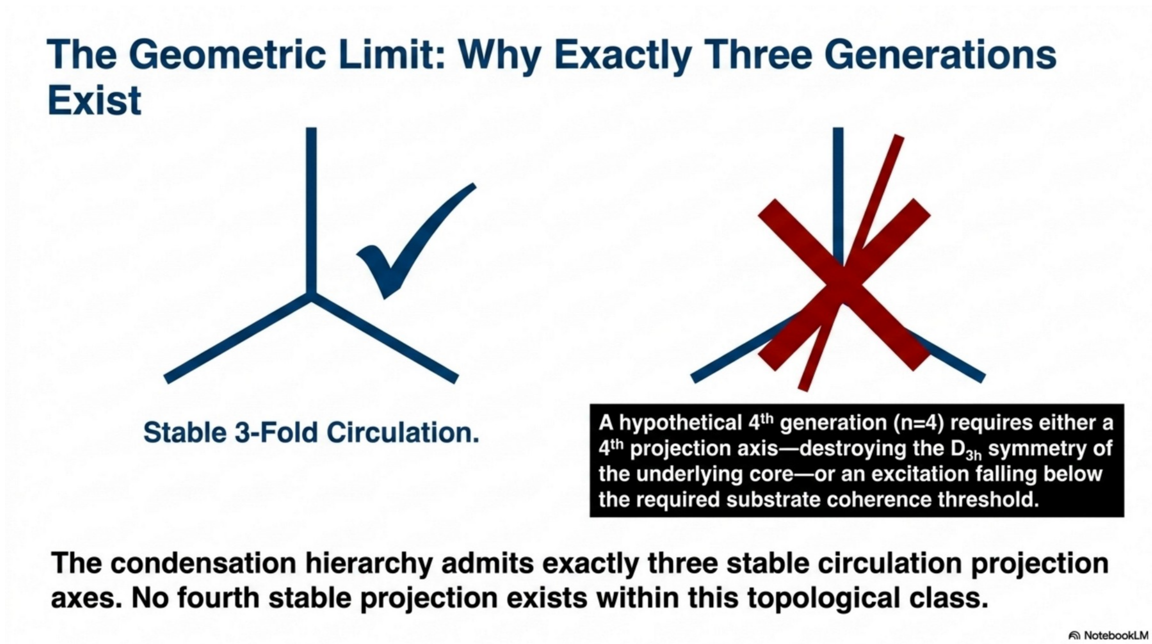


*Figure 5. The D3h condensation core admits exactly three stable circulation projection axes; a hypothetical fourth generation is excluded by this topology.*

The companion quark-sector analysis of the condensation framework identifies the analogous quark-generation cutoff quantitatively: a fourth up-type quark generation, projected by the same condensation hierarchy that produces the observed up, charm, and top quark masses, falls at approximately 0.067 MeV - far below the substrate coherence threshold required to sustain a stable condensation, and would not be expected to form a stable long-lived excitation. We propose that an analogous threshold applies to the charged lepton sector: the three existing D3h projection axes exhaust the stable three-fold circulation structure of the condensation core, and no fourth stable projection axis exists within the same topological class.

## 7. Discussion: Physical Interpretation and Anticipated Objections

### *7.1 Physical Interpretation*

Within the proposed interpretation, the three charged lepton masses are not three independent Standard Model parameters but three projections of a single internal circulation structure fixed by the D3h symmetry of the condensation core established independently in the companion papers of this series [1,2,3]. The Koide relation follows because the three-fold projection construction of equation (3) reproduces it for any phase, and the specific phase and mass scale are fixed by the same substrate parameters used throughout the series, not by parameters introduced specifically to fit the lepton masses.

### *7.2 "Is this merely a three-parameter fit dressed as a derivation?"*

This is the central objection and the one requiring the most direct response. Any three masses satisfying one exact constraint, such as equation (1), can be parametrised by two remaining free numbers, and it is always possible in principle to construct a geometric picture that reproduces three masses given two adjustable parameters. The response offered here is that the phase theta in equation (4) and the mass scale in Section 5.3 are not adjusted to fit the three lepton masses; they are computed from the three-unit core mode count Q = 2/3 of the 3+e topology [23,24] and the substrate-derived W boson mass m_W_vss of the companion electroweak derivation [3], both established independently without reference to the lepton sector at any step. The only directly measured lepton quantity carried over from lepton physics itself is the electron mass, used to fix the overall scale in Section 5.2, following the same practice used throughout this paper series of anchoring physical scales to one independently measured quantity. Given that anchor, the muon and tau masses in equations (6) and (7) are genuine outputs, not inputs, of the construction.

### *7.3 "Why should the three-core mode count Q = 2/3 equal the Koide ratio target 2/3?"*

This numerical coincidence between the three-core mode-counting fraction of Section 5.1 and the target value of the Koide ratio in equation (1) is a specific, structural claim of this paper: both quantities are proposed to trace to the same three-fold D3h symmetry of the condensation core, evaluated in two different contexts (the 3+e core mode counting of the companion papers [23,24] and lepton mass projection here). We regard this connection as a testable structural claim rather than an established fact; Section 8 identifies the observational consequences that would support or undermine it.

### *7.4 "Does this modify the Standard Model Yukawa sector?"*

No established prediction of the Standard Model lepton sector is affected. The proposal offers a geometric account of why the three Yukawa couplings take the specific ratios they do; it does not modify the Higgs mechanism, the renormalisation-group running of the lepton masses, or any confirmed electroweak precision measurement [4,13].

## 8. Falsifiable Predictions

The D3h condensation topology framework makes the following falsifiable predictions.

Prediction 1. No fourth charged lepton generation will be discovered at any accessible collider energy. Within the proposed framework, the D3h symmetry of the condensation core admits exactly three stable projection axes; the confirmed discovery of a fourth stable charged lepton would directly falsify the generation-count account of Section 6.

Prediction 2. As m_e, m_mu, m_tau, and m_W_vss are measured with improved precision, the relations of equations (5) through (9) are expected to remain consistent with the measured values to within their current precision or better. A statistically significant divergence, beyond what is attributable to higher-order corrections, would undermine the proposed structural connection between the electroweak and lepton mass sectors.

Prediction 3. The same D3h three-fold projection construction, applied independently to the quark sector using quark-sector mode-counting parameters analogous to Q = 2/3, provides a framework within which an analogous quark-sector relation may be investigated, though not necessarily with the same numerical target value given the additional colour degrees of freedom in the quark sector. This is identified as a specific direction for future work within this series rather than a result established in the present paper.

## 9. Conclusions

We have proposed a geometric origin for the Koide formula and the three charged lepton generations from the D3h point-group symmetry of a condensation topology, following the substrate framework established in prior work [1] and the condensation-geometry derivations of its companion papers [2,3,23,24]. Under this proposal, the three charged lepton masses arise as projections of a single internal circulation vector onto three axes separated by 120 degrees, fixed by the three-fold D3h symmetry of the first stable substrate condensation. The established three-fold projection parametrisation reproduces the Koide relation of equation (1) exactly, while the present work proposes that the D3h condensation topology provides its physical origin; the phase itself, and the overall mass scale, are fixed by the three-unit core mode-counting parameter Q = 2/3 of the 3+e topology [23,24] and the substrate-derived W boson mass m_W_vss of the companion electroweak derivation [3], both established independently, predicting the muon and tau masses to better than 0.02 percent once the independently established electron mass scale is specified, using a single substrate-derived phase parameter and one independently measured mass anchor.

We have proposed that the same D3h symmetry explains the observed count of exactly three charged lepton generations, since the condensation hierarchy admits exactly three stable circulation projection axes. Within the proposed framework, this interpretation requires no fine-tuning, no additional free parameters beyond those already fixed independently within the substrate framework of this series, and no modification to the confirmed structure of the Standard Model lepton sector. It requires only a proposed physical origin for a numerical relation that has been recognised as a precise and unexplained empirical regularity for more than four decades.

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